

Title	Complex Numbers – Trigonometric Form
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Date	20/1/2026

After being acquainted with the ability to deal with Complex Numbers in Cartesian form, I would like to introduce alternative ways to deal with Complex Numbers which is in its Trigonometric Form. It is very important to understand the following concepts before starting.

Please make sure your TI-84 Plus CE Graphing Calculator is in “Radian” Mode by following the instructions as follows:

Calculator Settings Check on TI-84 Plus CE or TI-84 Plus CE Python for Argument in Radian Mode as Follows:

Press [MODE] Button on TI-84 Plus CE

Check that the row “RADIAN DEGREE” has “RADIAN” mode highlighted on Graphing Calculator.

If the “RADIAN” Mode isn’t highlighted, highlight it by first pressing down arrow key until you reach “RADIAN DEGREE” row.

Once the cursor is on the row, press [←] key until the cursor jumps to “RADIAN” mode Press [ENTER] key and confirm “RADIAN” has been highlighted by pressing down arrow key once.

Once “RADIAN” is confirmed to be highlighted, press [CLEAR] button to exit.

The Argand Diagram (Complex Plane)

The Argand Diagram or the Complex Plane is a way to represent complex numbers where the horizontal axis represents real numbers and the vertical axis represent imaginary numbers, the point of origin of both real and vertical axis represents the zero complex number $z = 0 + 0i$.

Complex Numbers of any kind can be represented as shown below in the following example.

Example 1.

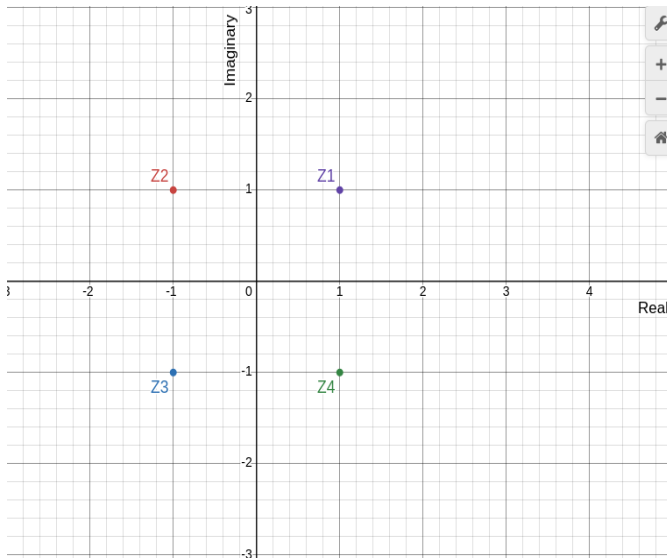
Draw an argand diagram and locate the following complex numbers on the complex plane.

(a) $Z_1 = 1 + i$

(b) $Z_2 = -1 + i$

(c) $Z_3 = -1 - i$

(d) $Z_4 = 1 - i$



As you can see, the process of using an argand diagram to represent any complex number is very similar to Coordinate Geometry as taught in Secondary School Elementary Mathematics, with a few modifications being made, namely the y -axis is to be reconfigured to serve as the imaginary axis and the x -axis being reconfigured to serve as the real axis.

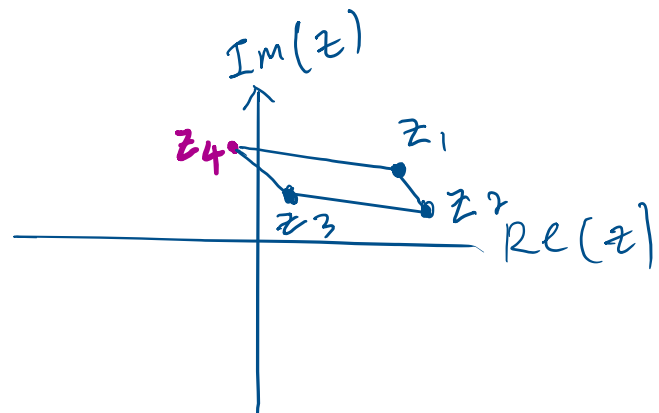
Example 2.

The vertices of a parallelogram $ABCD$ are represented by the complex numbers z_1, z_2, z_3 and z_4 respectively. Given that $z_1 = 2 + 3i, z_2 = 4 + i, z_3 = 1 + 2i$.

(a) On an argand diagram, sketch the parallelogram $ABCD$

(b) Find z_4 in Cartesian form

(a)



(b)

Using Complex Numbers Subtraction Principle

$$z_2 - z_1 = (4 + i) - (2 + 3i) = 4 + i - 2 - 3i = 2 - 2i$$

Since the line from z_1 to z_2 shares the same length as the line from z_4 to z_3 , the line from z_4 in the direction of z_3 can be represented as well by the complex number $2 - 2i$. As we are going from the direction of z_3 to z_4 , it should be represented by the complex number $-(2 - 2i)$ which is $-2 + 2i$.

Using Complex Numbers Addition Principle

$$z_4 = (1 + 2i) + (-2 + 2i) = z_4 = -1 + 4i$$

As you can see, the method of Adding or Subtracting complex numbers in a complex plane is very similar to the method of adding and subtracting in the topic of 2D Vectors as taught in Secondary Level Elementary Mathematics, replacing x and y displacement with real and imaginary displacement respectively.

Alternative way to represent Complex Numbers on a Complex Plane – Modulus Argument Form or the Trigonometric Form

In this section we are going to explore the alternative method of representing complex numbers on a complex plane, namely the Modulus Argument Form (A.K.A Trigonometric Form or the Polar Form).

Any complex number (excluding the zero complex number $0 + 0i$ as zero complex number has an undefined argument) can be expressed in the form $r(\cos(\theta) + i \sin \theta)$ with the letter r representing the modulus and θ representing the argument.

Modulus refers to the distance between the complex number and the point of origin. Given complex number z the modulus is typically represented by $|z|$ in A Level, given the complex number $a + bi$, the modulus is found using $|z| = \sqrt{a^2 + b^2}$.

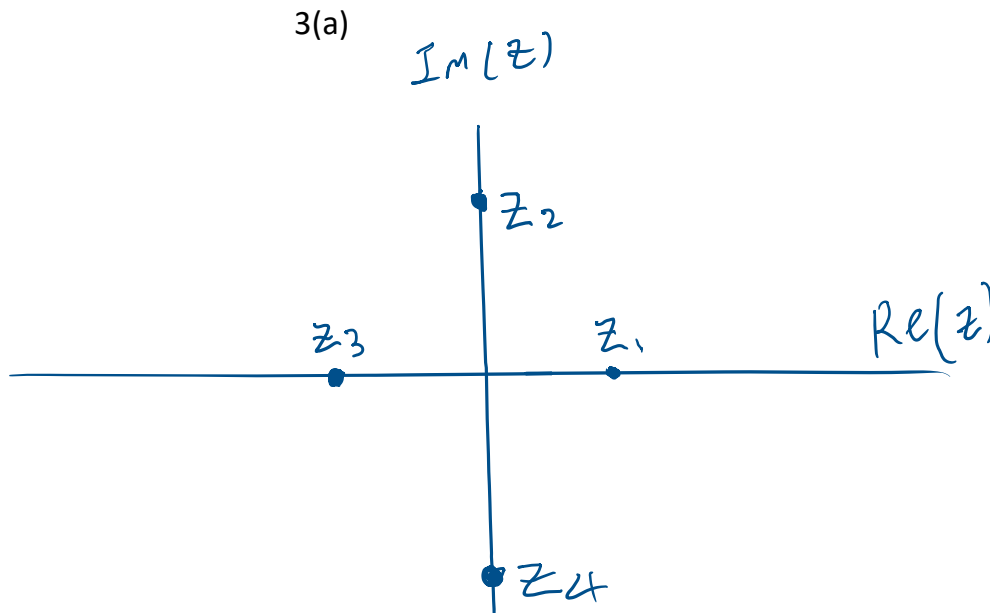
Argument refers to the angle of the complex number in relation to the real axis measured clockwise or anti clockwise. Given complex number z , the argument is denoted by $\arg(z)$. Rule of thumb here is that if the complex number is located in quadrant 1 or quadrant 2, the argument is measured clockwise with a positive angle. If the complex number is located in quadrant 3 and 4, the argument is measured anti-clockwise with a negative angle. (Note: All measurements of arguments are done in radians unless otherwise stated.)

Quadrant 2 To find argument of complex number in Quadrant 2, assuming complex number is in $a + bi$ form, we simply use $\pi - \tan^{-1}\left(\frac{b}{a}\right)$	Quadrant 1 To find argument of complex number in Quadrant 1, assuming complex number is in $a + bi$ form, we simply use $\tan^{-1}\left(\frac{b}{a}\right)$
Quadrant 3 To find argument of complex number in Quadrant 3, assuming complex number is in $a + bi$ form, we simply use $\tan^{-1}\left(\frac{b}{a}\right) - \pi$	Quadrant 4 To find argument of complex number in Quadrant 4, assuming complex number is in $a + bi$ form, we simply use $-\tan^{-1}\left(\frac{b}{a}\right)$

Example 3.

Given the following complex numbers: $z_1 = \sqrt{3}$, $z_2 = 5i$, $z_3 = -2$, $z_4 = -4i$

- On a single argand diagram locate z_1, z_2, z_3 and z_4 .
- Hence express z_1, z_2, z_3 and z_4 in modulus argument form.



3(b)

$$z_1 = \sqrt{3} + 0i$$

$$|z_1|^2 = (\sqrt{3})^2 + (0)^2 = 3$$

$$|z_1| = \sqrt{3}$$

$$\arg(z_1) = 0$$

$$\sqrt{3}(\cos(0) + i \sin(0))$$

$$z_2 = 0 + 5i$$

$$|z_2|^2 = 0^2 + 5^2 = 5^2$$

$$|z_2| = \sqrt{5^2} = 5$$

$$\arg(z_2) = \frac{\pi}{2}$$

$$5\left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right)$$

$$z_3 = -2 + 0i$$

$$|z_3|^2 = (-2)^2 + (0)^2 = 2$$

$$\arg(z_3) = \pi$$

$$2(\cos(\pi) + i \sin(\pi))$$

$$z_4 = 0 - 4i$$

$$|z_4|^2 = 0^2 + (-4)^2$$

$$|z_4| = 4$$

$$\arg(z_4) = -\frac{\pi}{2}.$$

$$4\left(\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)\right)$$

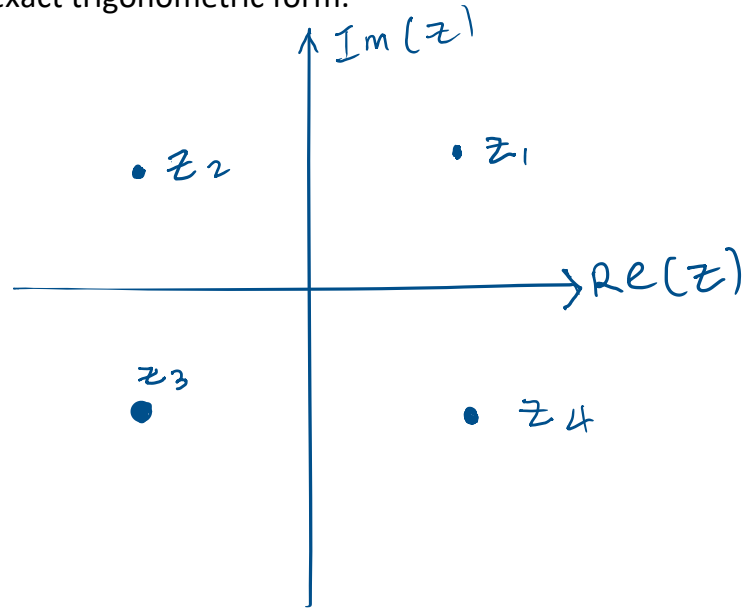
Example 4.

Given the following complex numbers

$$z_1 = 1 + i, z_2 = -1 + i, z_3 = -1 - i, z_4 = 1 - i$$

(a) Using a single Argand Diagram, locate z_1, z_2, z_3 and z_4

(b) Hence express z_1, z_2, z_3 and z_4 in exact trigonometric form.



4(b)

Since z_1 is within quadrant 1, we shall measure the argument **clockwise** from the positive real axis, which would give us the following:

$$\arg(z_1) = \tan^{-1}\left(\frac{1}{1}\right) = \left(\frac{\pi}{4}\right)$$

$$|z_1| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$z_1 = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$$

Since z_2 is within quadrant 2, we shall measure the argument **clockwise** from the positive real axis, which would give us the following:

$$\arg(z_2) = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$|z_2| = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$z_2 = \sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$$

Since z_3 is within quadrant 3, we shall measure the argument **anti-clockwise** from the positive real axis, which would give us the following:

$$\arg(z_3) = \tan\left(\frac{1}{1}\right) - \pi = -\frac{3\pi}{4}$$

$$|z_3| = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

$$z_3 = \sqrt{2} \left(\cos\left(-\frac{3\pi}{4}\right) + i \sin\left(-\frac{3\pi}{4}\right) \right)$$

Since z_4 is within quadrant 4, we shall measure the argument **anti-clockwise** from the positive real axis

$$\arg(z_4) = -\tan\left(\frac{1}{1}\right) = -\frac{\pi}{4}$$

$$|z_4| = \sqrt{a^2 + b^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$z_4 = \sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)$$

Graphing Calculator Instructions for TI-84 Plus CE to Verify Values of Modulus and Argument

Example complex number shown below is: $3 - 4i$

Finding Modulus with TI-84 Plus CE

Press [MATH] Button

Press [→] Button Twice So That Cursor Jumps to "CMPLX"

Press [5] and the display will show two vertical strokes with empty placeholder within the two vertical strokes.

From within the two vertical strokes, key in the complex number that you want to find the modulus for, using the above example, we key in $3 - 4i$ and press [ENTER] we get a modulus value of 5.

Finding Argument with TI-84 Plus CE

Press [MATH] Button

Press [→] Button Twice So That Cursor Jumps to “CMPLX”

Press [4] and the word “angle(“ should appear.

Following the example complex number from above, the complex number I will key in after the opening bracket is $3 - 4i$ and press [ENTER] and you should get the argument in radians which is -0.927295218 .

Multiplication and Division of Complex Number in Trigonometric Form

Given two complex numbers z_1 and z_2 are being written in Trigonometric Form, as shown below.

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$$

$$z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

Product of the two complex numbers as shown above can be expressed using the following method.

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

In short, if you are given a task that involves multiplication of complex numbers in polar form, you multiply the modulus as derived from both complex numbers together and add the argument as derived from both complex numbers.

Given two complex numbers z_1 and z_2 are being written in Trigonometric Form, as shown below.

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$$

$$z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

The value of $\frac{z_1}{z_2}$ can be written as follows:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$$

In short, if you are given a task that involves division of complex numbers in polar form, you divide the modulus as derived from both complex numbers and subtract the argument as derived from both complex numbers.

Important Concept of Principal Argument:

The value of $\arg(zw) = (\theta_1 + \theta_2)$ or $\arg\left(\frac{z}{w}\right) = (\theta_1 - \theta_2)$ sometimes can exceed or fall below the values of between $-\pi$ and π . If such events occur, you have to add or subtract multiples of 2π to obtain the principal argument of the product or quotient in question.

Example 5.

Given that $z = 2 + 2\sqrt{3}i$ and $w = -5\sqrt{3} + 5i$

(a) Express both z and w in trigonometric form

(b) Hence express zw and $\frac{z}{w}$ in trigonometric form

5(a)

Converting z and w into trigonometric form, we get the following:

$$\arg(z) = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

$$z = 4\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$

$$\arg(w) = \pi - \tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) = \frac{5\pi}{6}$$

$$|w| = \sqrt{(5\sqrt{3})^2 + 5^2} = 10$$

$$w = 10\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$$

5(b)

$$zw = 10(4)\left(\cos\left(\frac{5\pi}{6} + \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{6} + \frac{\pi}{3}\right)\right)$$

$$zw = 40\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$$

Since $\frac{7\pi}{6} > \pi$, we subtract 2π from the argument to get the principal argument value which is $-\frac{5\pi}{6}$

$$\text{Hence } zw = 40\left(\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right)$$

$$\frac{z}{w} = \frac{4}{10} \left(\cos \left(\frac{\pi}{3} - \frac{5\pi}{6} \right) + i \sin \left(\frac{\pi}{3} - \frac{5\pi}{6} \right) \right)$$

$$\frac{z}{w} = \frac{4}{10} \left(\cos \left(-\frac{\pi}{2} \right) + i \sin \left(-\frac{\pi}{2} \right) \right)$$